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from: *Seth King, 8881* SETH KING (Affiliate) Digitally signed by SETH KING (Affiliate) Date: 2025.04.09 14:11:35 -06'00'

subject: *Constructed Cumulative Distributions for Approximation of Truncated Normal Distributions*

INTRODUCTION

The WIPP Performance Assessment (PA) code LHS samples values from input parameter distributions using an efficient Latin Hypercube Sampling algorithm. Latin hypercube sampling reduces the minimum number of sample vectors required to obtain reasonable estimates for various statistical properties of a distribution (WIPP PA, 2021). One of the distributions the LHS code can sample from is a truncated normal distribution.

When sampling from a truncated normal distribution, the LHS code defines the truncated normal from only the user input minimum and maximum points on the distribution. LHS assumes the minimum value is at the 0.01 quantile and maximum value is at the 0.99 quantile of the normal distribution (WIPP PA, 2021; Tierney, 1996)[1]. Given user input a for the minimum value and b for the maximum value, the parameters of the truncated normal distribution can be expressed as:

$$E(X) = \mu = \frac{(a+b)}{2} \tag{1}$$

$$V(X) = \sigma^2 = \left(\frac{b-a}{2*z}\right)^2 \tag{2}$$

Where z is the two-tailed Z-score giving the inner 98% probability. That is, z such that:

$$P(-z < X < z) = 0.98 \tag{3}$$

giving

$$z = 2.326 \tag{4}$$

[1] Note that Tierney (1996) uses a Z-score of 3.09 for the 0.001 and 0.999 quantiles.

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That is, LHS can only sample from normal distributions truncated at plus and minus 2.326 standard deviations from the mean.

LHS allows a cumulative distribution specification where the user inputs discrete points to define the cumulative distribution function (CDF). A cumulative distribution can be used for input parameters with truncated normal distributions where the truncation does not happen at 2.326 standard deviations from the mean.

For the CRA-2026, there are three such parameters, PCS_T2:POR2PERM, WAS_AREA:BRUCITEC, and WAS_AREA:BRUCITES. Using the python script presented in Appendix A, the CDF values at 101 equally spaced probabilities are found for each parameter to construct a cumulative distribution approximation of the specified truncated normal distribution.

PCS_T2:POR2PERM

Camphouse et al. (2012) defined the relationship between porosity and permeability in a run-of-mine salt panel closure system during the secondary time duration using a unitless parameter PCS_T2:POR2PERM with a normal distribution of mean 0 and standard deviation of 0.86, truncated at ±2 standard deviations. Camphouse (2012) recommended using this distribution in the WIPP PA calculations. The cumulative distribution data to approximate this truncated normal CDF are given in Table 1.

Table 1 – Cumulative Distribution Data for PCS_T2:POR2PERM

Order Index	Value	Probability
0	-1.72000	0.00
1	-1.58935	0.01
2	-1.48756	0.02
3	-1.40315	0.03
4	-1.33043	0.04
5	-1.26615	0.05
6	-1.20825	0.06
7	-1.15536	0.07
8	-1.10651	0.08
9	-1.06099	0.09
10	-1.01827	0.10
11	-0.97792	0.11
12	-0.93961	0.12
13	-0.90309	0.13
14	-0.86812	0.14
15	-0.83454	0.15
16	-0.80218	0.16
17	-0.77092	0.17
18	-0.74064	0.18
19	-0.71126	0.19
20	-0.68269	0.20
21	-0.65484	0.21

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22	-0.62767	0.22
23	-0.60112	0.23
24	-0.57512	0.24
25	-0.54964	0.25
26	-0.52463	0.26
27	-0.50006	0.27
28	-0.47589	0.28
29	-0.45209	0.29
30	-0.42863	0.30
31	-0.40548	0.31
32	-0.38263	0.32
33	-0.36004	0.33
34	-0.33770	0.34
35	-0.31558	0.35
36	-0.29368	0.36
37	-0.27196	0.37
38	-0.25041	0.38
39	-0.22902	0.39
40	-0.20776	0.40
41	-0.18664	0.41
42	-0.16563	0.42
43	-0.14471	0.43
44	-0.12388	0.44
45	-0.10313	0.45
46	-0.08243	0.46
47	-0.06178	0.47
48	-0.04117	0.48
49	-0.02058	0.49
50	0.00000	0.50
51	0.02058	0.51
52	0.04117	0.52
53	0.06178	0.53
54	0.08243	0.54
55	0.10313	0.55
56	0.12388	0.56
57	0.14471	0.57
58	0.16563	0.58
59	0.18664	0.59
60	0.20776	0.60
61	0.22902	0.61
62	0.25041	0.62
63	0.27196	0.63
64	0.29368	0.64
65	0.31558	0.65
66	0.33770	0.66
67	0.36004	0.67
68	0.38263	0.68
69	0.40548	0.69
70	0.42863	0.70
71	0.45209	0.71

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72	0.47589	0.72
73	0.50006	0.73
74	0.52463	0.74
75	0.54964	0.75
76	0.57512	0.76
77	0.60112	0.77
78	0.62767	0.78
79	0.65484	0.79
80	0.68269	0.80
81	0.71126	0.81
82	0.74064	0.82
83	0.77092	0.83
84	0.80218	0.84
85	0.83454	0.85
86	0.86812	0.86
87	0.90309	0.87
88	0.93961	0.88
89	0.97792	0.89
90	1.01827	0.90
91	1.06099	0.91
92	1.10651	0.92
93	1.15536	0.93
94	1.20825	0.94
95	1.26615	0.95
96	1.33043	0.96
97	1.40315	0.97
98	1.48756	0.98
99	1.58935	0.99
100	1.72000	1.00

WAS_AREA:BRUCITEC

Nemer (2025) defined the MgO inundated hydration rate in ERDA-6 brine (WAS_AREA:BRUCITEC) as a truncated normal distribution, in units of moles/(kg * s) with mean of 8.2×10^{-8}, standard deviation of 2.0×10^{-7}, minimum of 7.6×10^{-8}, and maximum of 8.8×10^{-8}. This is a truncation at three standard deviations from the mean. The cumulative distribution data to approximate this truncated normal CDF are given in Table 2.

Table 2 – Cumulative Distribution Data for WAS_AREA:BRUCITEC

Order Index	Value	Probability
0	7.6000E-08	0.00
1	7.7441E-08	0.01
2	7.7945E-08	0.02
3	7.8275E-08	0.03
4	7.8527E-08	0.04
5	7.8734E-08	0.05
6	7.8910E-08	0.06
7	7.9066E-08	0.07

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8	7.9205E-08	0.08
9	7.9332E-08	0.09
10	7.9449E-08	0.10
11	7.9558E-08	0.11
12	7.9660E-08	0.12
13	7.9757E-08	0.13
14	7.9848E-08	0.14
15	7.9935E-08	0.15
16	8.0019E-08	0.16
17	8.0099E-08	0.17
18	8.0176E-08	0.18
19	8.0250E-08	0.19
20	8.0323E-08	0.20
21	8.0393E-08	0.21
22	8.0461E-08	0.22
23	8.0527E-08	0.23
24	8.0592E-08	0.24
25	8.0655E-08	0.25
26	8.0717E-08	0.26
27	8.0778E-08	0.27
28	8.0838E-08	0.28
29	8.0897E-08	0.29
30	8.0954E-08	0.30
31	8.1011E-08	0.31
32	8.1067E-08	0.32
33	8.1123E-08	0.33
34	8.1177E-08	0.34
35	8.1232E-08	0.35
36	8.1285E-08	0.36
37	8.1338E-08	0.37
38	8.1391E-08	0.38
39	8.1443E-08	0.39
40	8.1495E-08	0.40
41	8.1546E-08	0.41
42	8.1597E-08	0.42
43	8.1648E-08	0.43
44	8.1699E-08	0.44
45	8.1749E-08	0.45
46	8.1800E-08	0.46
47	8.1850E-08	0.47
48	8.1900E-08	0.48
49	8.1950E-08	0.49
50	8.2000E-08	0.50
51	8.2050E-08	0.51
52	8.2100E-08	0.52
53	8.2150E-08	0.53
54	8.2200E-08	0.54
55	8.2251E-08	0.55
56	8.2301E-08	0.56
57	8.2352E-08	0.57

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58	8.2403E-08	0.58
59	8.2454E-08	0.59
60	8.2505E-08	0.60
61	8.2557E-08	0.61
62	8.2609E-08	0.62
63	8.2662E-08	0.63
64	8.2715E-08	0.64
65	8.2768E-08	0.65
66	8.2823E-08	0.66
67	8.2877E-08	0.67
68	8.2933E-08	0.68
69	8.2989E-08	0.69
70	8.3046E-08	0.70
71	8.3103E-08	0.71
72	8.3162E-08	0.72
73	8.3222E-08	0.73
74	8.3283E-08	0.74
75	8.3345E-08	0.75
76	8.3408E-08	0.76
77	8.3473E-08	0.77
78	8.3539E-08	0.78
79	8.3607E-08	0.79
80	8.3677E-08	0.80
81	8.3750E-08	0.81
82	8.3824E-08	0.82
83	8.3901E-08	0.83
84	8.3981E-08	0.84
85	8.4065E-08	0.85
86	8.4152E-08	0.86
87	8.4243E-08	0.87
88	8.4340E-08	0.88
89	8.4442E-08	0.89
90	8.4551E-08	0.90
91	8.4668E-08	0.91
92	8.4795E-08	0.92
93	8.4934E-08	0.93
94	8.5090E-08	0.94
95	8.5266E-08	0.95
96	8.5473E-08	0.96
97	8.5725E-08	0.97
98	8.6055E-08	0.98
99	8.6559E-08	0.99
100	8.8000E-08	1.00

WAS_AREA:BRUCITES

Nemer (2025) defined the MgO inundated hydration rate in GWB brine (WAS_AREA:BRUCITES) as a truncated normal distribution with mean of 1.4×10^{-7} mol/(kg * s), standard deviation of 5.0×10^{-8} mol/(kg * s), minimum of 0 mol/(kg * s), and maximum of 2.9

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$\times 10^{-7}$ mol/(kg * s). This is a truncation at three standard deviations from the mean for the maximum and 2.8 standard deviations from the mean for the minimum, giving a non-symmetric truncated distribution. The cumulative distribution data to approximate this truncated normal CDF are given in Table 3.

Table 3 – Cumulative Distribution Data for WAS_AREA:BRUCITES

Order Index	Value	Probability
0	0.000E+00	0.00
1	2.795E-08	0.01
2	3.975E-08	0.02
3	4.769E-08	0.03
4	5.382E-08	0.04
5	5.888E-08	0.05
6	6.322E-08	0.06
7	6.705E-08	0.07
8	7.049E-08	0.08
9	7.363E-08	0.09
10	7.653E-08	0.10
11	7.923E-08	0.11
12	8.177E-08	0.12
13	8.416E-08	0.13
14	8.643E-08	0.14
15	8.860E-08	0.15
16	9.067E-08	0.16
17	9.266E-08	0.17
18	9.458E-08	0.18
19	9.644E-08	0.19
20	9.823E-08	0.20
21	9.998E-08	0.21
22	1.017E-07	0.22
23	1.033E-07	0.23
24	1.049E-07	0.24
25	1.065E-07	0.25
26	1.081E-07	0.26
27	1.096E-07	0.27
28	1.111E-07	0.28
29	1.125E-07	0.29
30	1.140E-07	0.30
31	1.154E-07	0.31
32	1.168E-07	0.32
33	1.182E-07	0.33
34	1.195E-07	0.34
35	1.209E-07	0.35
36	1.222E-07	0.36
37	1.236E-07	0.37
38	1.249E-07	0.38
39	1.262E-07	0.39
40	1.275E-07	0.40

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41	1.287E-07	0.41
42	1.300E-07	0.42
43	1.313E-07	0.43
44	1.326E-07	0.44
45	1.338E-07	0.45
46	1.351E-07	0.46
47	1.363E-07	0.47
48	1.376E-07	0.48
49	1.388E-07	0.49
50	1.401E-07	0.50
51	1.413E-07	0.51
52	1.426E-07	0.52
53	1.438E-07	0.53
54	1.451E-07	0.54
55	1.463E-07	0.55
56	1.476E-07	0.56
57	1.489E-07	0.57
58	1.501E-07	0.58
59	1.514E-07	0.59
60	1.527E-07	0.60
61	1.540E-07	0.61
62	1.553E-07	0.62
63	1.566E-07	0.63
64	1.579E-07	0.64
65	1.593E-07	0.65
66	1.606E-07	0.66
67	1.620E-07	0.67
68	1.634E-07	0.68
69	1.648E-07	0.69
70	1.662E-07	0.70
71	1.676E-07	0.71
72	1.691E-07	0.72
73	1.706E-07	0.73
74	1.721E-07	0.74
75	1.737E-07	0.75
76	1.752E-07	0.76
77	1.769E-07	0.77
78	1.785E-07	0.78
79	1.802E-07	0.79
80	1.820E-07	0.80
81	1.838E-07	0.81
82	1.856E-07	0.82
83	1.876E-07	0.83
84	1.896E-07	0.84
85	1.917E-07	0.85
86	1.938E-07	0.86
87	1.961E-07	0.87
88	1.985E-07	0.88
89	2.011E-07	0.89
90	2.038E-07	0.90

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91	2.067E-07	0.91
92	2.099E-07	0.92
93	2.134E-07	0.93
94	2.173E-07	0.94
95	2.217E-07	0.95
96	2.268E-07	0.96
97	2.331E-07	0.97
98	2.414E-07	0.98
99	2.540E-07	0.99
100	2.900E-07	1.00

OTHER PARAMETERS

Clayton (2013) defined the MgO humid hydration rate (WAS_AREA:BRUCITEH) with a normal distribution truncated at 2.326 standard deviations from the mean. As this distribution fits the LHS assumption about a truncated normal distribution, it does not need to be approximated with a cumulative distribution. Camphouse (2012) defined the same distribution for the porosity to permeability relationship for the panel closure system during the third time period (PCS_T3:POR2PERM) as the second time period (PCS_T2:POR2PERM), however the sampled value for the second time period is used for the third time period and PCS_T3:POR2PERM is not used in the PA calculation. There are no other parameters being used in the CRA-2026 PA calculations that are defined with a normal or a lognormal distribution.

REFERENCES

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APPENDIX A – CALCULATION OF CUMULATIVE DATA

The following python script was used to generate the cumulative distribution data to approximate the specified truncated normal distributions.

```
import numpy as np
from scipy.stats import truncnorm
import matplotlib.pyplot as plt
import pandas as pd

brucites = {'mean': 1.4e-7,
            'sd': 5e-8,
            'max': 2.9e-7,
            'min': 0,
            'mat': 'WAS_AREA'}
brucitec = {'mean': 8.2e-8,
            'sd': 0.2e-8,
            'max': 8.8e-8,
            'min': 7.6e-8,
            'mat': 'WAS_AREA'}
por2perm = {'mean': 0,
            'sd': 0.86,
            'max': 1.72,
            'min': -1.72,
            'mat': 'PCS_T2'}

dfs = {}
for dname, dist in zip(['brucitec', 'brucites', 'por2perm'],
                       [brucitec, brucites, por2perm]):

    a = (dist['min'] - dist['mean']) / dist['sd']
    b = (dist['max'] - dist['mean']) / dist['sd']

    probs = np.linspace(0, 1, 101)
    values = truncnorm.ppf(probs, a, b, loc=dist['mean'], scale=dist['sd'])
    dfs[dname] = pd.DataFrame({'Value': values,
                               'Probability': probs})
    fig, ax = plt.subplots(figsize=(8,6))
    ax.plot(values, probs)
    ax.set_xlabel('Hydration Rate ($mols/(kg * s)$)')
    ax.set_ylabel('Probability')
    ax.grid()
    ax.set_title(f"{dist['mat']}:{dname.upper()}")
    fig.tight_layout()
    fig.savefig(f'{dname}.png', dpi=300)
```

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```
with pd.ExcelWriter('Constructed_Normal_Distributions.xlsx') as ex_writer:
    for dname in dfs:
        df = dfs[dname]
        df.to_excel(ex_writer, sheet_name=dname)

for dname in dfs:
    df = dfs[dname]
    mean = 0
    var = 0
    for i in range(1, len(df)):
        dp = df.loc[i, 'Probability'] - df.loc[i-1, 'Probability']
        avg = (df.loc[i, 'Value'] + df.loc[i-1, 'Value']) / 2
        mean += dp * avg
        v3 = (df.loc[i, 'Value']*df.loc[i, 'Value'] +
              df.loc[i, 'Value']*df.loc[i-1, 'Value'] +
              df.loc[i-1, 'Value']*df.loc[i-1, 'Value']) / 3
        var += dp * v3
    var = var - mean*mean
    sd = np.sqrt(var)
    print(f'{dname} - Mean:{mean:.4e}, SD:{sd:.4e}')
```

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